Modeling Cell Layers on Complex Surfaces Using Constrained Voronoi Diagrams

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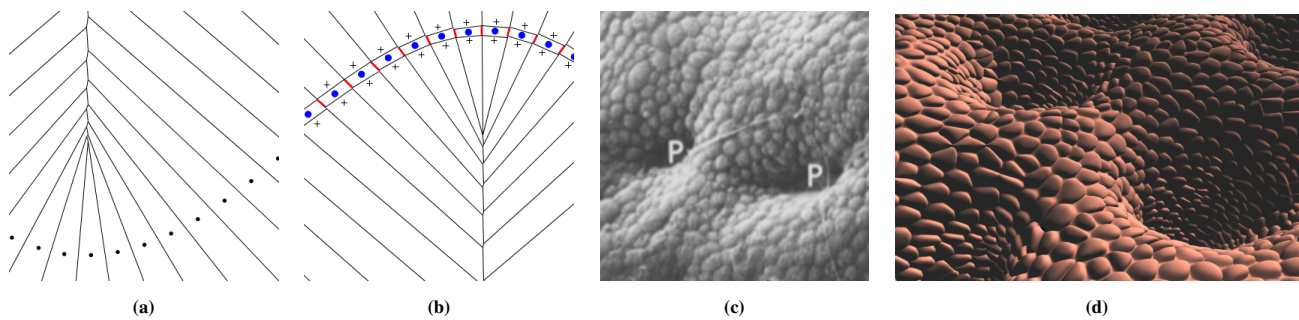
Figure 1: *Points on a sine wave illustrate which Voronoi regions share faces before (a) and after (b) the addition of augment points. Scanning electron microscope images (c) of gastric pits in human stomach lining [Mackercher et al. 1978] are closely mimicked by an extruded and smoothed mesh of a constrained Voronoi diagram (d) generated using my technique.*

Abstract

Voronoi diagrams, found in the morphology of many biological structures, produce natural-looking networks of cells in two dimensions. But creating three dimensional models of cell layers in biological tissue is difficult when the tissue has a complex shape. I present a new method that constrains a Voronoi diagram to an arbitrary surface. Unlike other approaches, my method does not require computationally expensive boundary intersection calculations.

1 Contribution

A two-dimensional illustration of a layer of cells can be produced directly from the Voronoi diagram of a set of points or sites distributed randomly inside a bounding shape. But despite the ubiquity of Voronoi diagrams in computer graphics applications, only a few efforts have been made to extend Voronoi-based models of cell layers to three dimensions. A significant problem is that regions located on the boundary of a Voronoi diagram can have edges that extend very far from it. In two dimensions, most Voronoi regions are on the interior and are unaffected by the boundary conditions. In three dimensions, however, the regions on the boundary form the visible surface of the shape. One approach to this problem [Yan et al. 2010] is to generate a Voronoi diagram of sites placed throughout the entire volume enclosed by the surface and derive a network of cells from the intersections of the Voronoi diagram with the surface. My approach restricts site placement to the surface and uses additional sites to produce a Voronoi diagram that can be constrained using less computationally expensive methods.

2 Technical Approach

First, points are placed on every triangular face of the surface using the triangle point picking method, with the number of points generated for each triangle directly proportional to its area. Generating each point in this way allows us to calculate a normal for each point using the barycentric average of the normals of the triangle's vertices.

Next, additional Voronoi sites, called augment sites, are placed on either side of the surface (Figure 1b), effectively insulating the orig-

inal Voronoi sites from all but their immediate neighbors. It is here that the barycentric normal of each Voronoi site is used to calculate the displacement of its pair of augment sites. The magnitude of the displacement must be chosen to ensure that for each original Voronoi site, its augment sites will be closer than any other sites. This method exploits the properties of Voronoi diagrams and eliminates the need to search the boundary faces for intersections.

The Voronoi diagram of the augmented set of sites is calculated using the QHULL algorithm. Only faces shared by sites in the original point set before augmentation are recorded. All faces in the regions of augment sites and between an original site and an augment site are ignored.

Once the 3D Voronoi diagram is generated from the augmented point set, several steps need to be taken to transfer the diagram into a network of cells (one polygon for each Voronoi region) on the original surface. The list of faces each Voronoi region shares with other regions is used to find the vertices describing a polygon on the original surface. Because these vertices are not in order, I employ a heuristic approximation of the clockwise order of the points around the Voronoi site to form a convex polygon.

While a Centroidal Voronoi Tesselation (CVT) would not look organic, minimizing the distance between each site and the centroid of its region improves the realism of the output. To relax the diagram, I added a function to complete a few iterations of Lloyds algorithm. Extruding the faces of the relaxed diagram produces a mesh that can be smoothed (Figure 1d) to closely resemble reference images of real cell layers.

References

MACKERCHER, P. A., IVEY, K. J., BASKIN, W. N., AND KRAUSE, W. J. 1978. A scanning electron microscopic study of normal human oxyntic mucosa using blunt dissection and freeze fracture. *Am J Dig Dis 23*, 5, 449–59.

YAN, D.-M., WANG, W., LÉVY, B., AND LIU, Y. 2010. Efficient computation of 3d clipped voronoi diagram. In *Proceedings of the 6th international conference on Advances in Geometric Modeling and Processing*, Springer-Verlag, Berlin, Heidelberg, GMP'10, 269–282.

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